THERMAL RESISTANCE OF MULTICONTACT PACKETS

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A method of calculating the thermal resistance of multicontact packets is proposed. A comparison of the calculation with the experiment is given.

Heat transfer in multicontact packets of plates under vacuum conditions is accomplished mainly by heat conduction through the contact areas of the plate surfaces.

Owing to the greater heat conduction of the contact areas in comparison with the conduction between the contacts, contraction of the lines of thermal fluxes toward the areas occurs, and the isotherms are curved and have a complex appearance near the contacts ([1], p. 15).

We will present a schematic pattern of heat transfer in a multicontact packet in which the thickness of each plate is greater than the sum of the height of the microprojections of the upper and lower contacts (by two and more orders of magnitude). This pattern can be depicted as shown in Fig. 1 depending on the number of areas in contact.

The contact surfaces as a whole form the actual contact area of the plates and act as resistances connected in parallel. Each plate of the packet can be regarded as consisting of a set of elements with adiabatic surfaces having two contact surfaces each. Then the total thermal resistance of the plate can be found by analogy with the determination of the total electrical resistance on the basis of the condition of parallel-acting resistances.

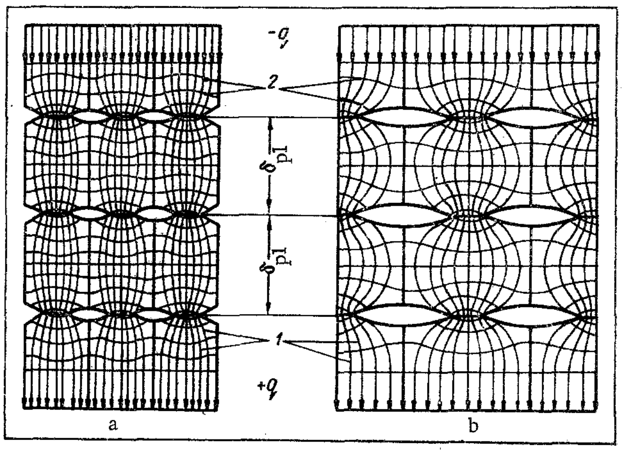

Fig. 1. Schematic pattern of the flow of heat in a multicontact packet of plates with δ_{pl} > sum of the heights of the microprojections of the upper and lower contacts: a, b) for a large and small number of contacts on the surface, respectively; 1) lines of thermal flux; 2) isotherms.

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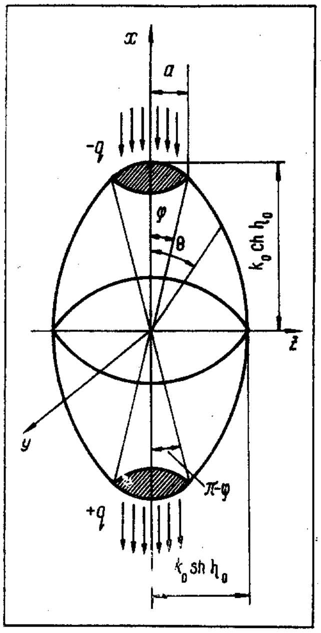

Fig. 2. Calculation model of heat transfer in an ellipsoid with two contact areas.

As an element we take an ellipsoid of revolution with semimajor and semiminor axes $h_e = \delta_{pl}/2$ and $b_e = \varkappa h_e$. We will determine the temperature field of such ellipsoid, considering a, h_e, b_e, q, and $\bar{\lambda}$ are given and using certain assumptions made when deriving the formula for the temperature field in a sphere with two contact surfaces [2], namely, we assume that the contact surfaces are not flat but are a part of the surface of the ellipsoid. Then in view of the smallness of the area of the contact spot we can consider that $a/h_e = \sin\varphi$.

A two-parameter family of ellipsoids of revolution about the x axis (parameters k, η) with $b_e < h_e$ in a rectangular coordinate system (Fig. 2) is described by a system of three equations (for ellipsoids of revolution with $b_e > h_e$ it is necessary to change the places of the x and y axes):

$$x = k\cos\theta \operatorname{ch}\eta,$$
$$y = k\sin\theta \operatorname{sh}\eta\cos\omega,$$
$$z = k\sin\theta \operatorname{sh}\eta\sin\omega.$$

We introduce the notations:

$$\xi = \operatorname{ch}\eta, \quad \nu = \cos\theta.$$

Then we can obtain the formula for the temperature field of an ellipsoid with two contact surfaces, taking into account axial symmetry, by integrating the Laplace equation written in ellipitical coordinates [3] for the following boundary conditions:

$$\frac{\partial}{\partial\xi}\left[(\xi^2-1)\frac{\partial T}{\partial\xi}\right]+\frac{\partial}{\partial\nu}\left[(1-\nu^2)\frac{\partial T}{\partial\nu}\right]=0 \tag{1}$$

for the following boundary conditions:

$$\left.\frac{\partial T}{\partial n}\right|_{P_e \in S_e} = f(\theta) = \begin{cases} -\dfrac{q}{\lambda}, & 0<\theta<\varphi \\ 0, & \varphi<\theta<\pi-\varphi \\ +\dfrac{q}{\lambda}, & \pi-\varphi<\theta<\pi. \end{cases} \tag{2}$$

Since the product of two Legendre's polynomials of the same order as a function of the variables ξ and ν is the particular solution of Eq. (1), we write the general solution of (1) in the following form:

$$T(\xi,\nu)=\sum_{k=0}^{\infty} a_k P_k(\xi) P_k(\nu) \quad (\xi \leqslant \xi_0). \tag{3}$$

Using an elliptical coordinate system, we can write the boundary conditions (2)

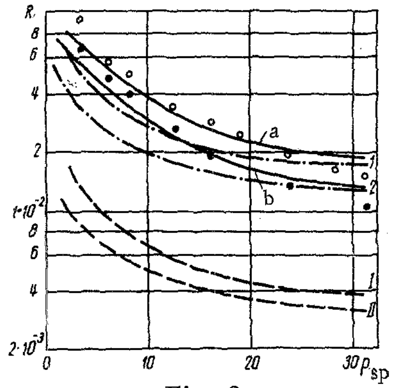

Fig. 3

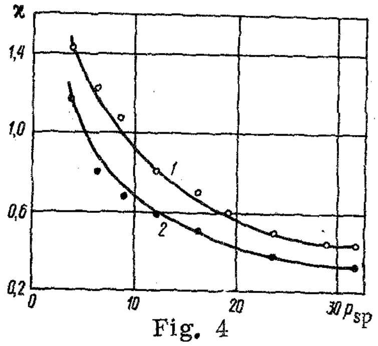

Fig. 4

Fig. 3. Thermal resistance R (deg·m^2/W) as a function of the specific force p_{sp} (MN/m^2) for multicontact packets of plates with δ_{pl} = 0.1 mm, n = 320 units and δ_{pl} = 0.2 mm, n = 175 units of stainless steel Kh18N10T-M: I, II) data of calculation by theoretical model in [1]; 1, 2) the same, in [5]; a, b) data of calculation by Eq. (7) for $\varkappa$ = 0.8 and 0.7, respectively; circles: experimental data.

Fig. 4. Coefficient of contraction of the ellipsoid $\varkappa = b_e/h_e$ as a function of the specific force p_{sp} (MN/m^2) for packets of plates of stainless steel Kh18N10T-M: 1) δ_{pl} = 0.1 mm, n = 320 units; 2) δ_{pl} = 0.2 mm, n = 175 units.

$$\frac{\sqrt{\xi^2-1}}{k\sqrt{\xi^2-\nu^2}}\left.\frac{\partial T}{\partial \xi}\right|_{\substack{k=k_0\\ \xi=\xi_0}} = f(\theta). \tag{4}$$

We rewrite Eq. (3) in the following manner:

$$T(\xi, \nu) = a_0 + \sum_{k=1}^{\infty} a_k P_k(\xi) P_k(\nu) \quad (\xi \leqslant \xi_0) \tag{5}$$

and find the coefficients a_0 and a_k.

To determine the coefficients a_k (k = 1, 2, 3, . . .) we use boundary conditions (4), using in so doing the orthogonality of Legendre's polynomials on the interval [−1, 1]. For this purpose we differentiate (5) with respect to ξ, multiply by $\sqrt{(\xi^2-1)}/k\sqrt{\xi^2-\nu^2}$, and, taking into account (4) and that $\|P_k(x)\|^2 = 2/(2k+1)$, we obtain

$$a_k = \frac{(2k+1)k_0 q}{2\sqrt{\xi_0^2-1}\,P_k'(\xi_0)\,\lambda}\left(\int_{\pi-\varphi}^{\pi} - \int_0^{\varphi}\right) P_k(\cos\theta)\sqrt{\xi_0^2-\cos^2\theta}\,\sin\theta\,d\theta$$

$$= \frac{(2k+1)k_0 q\,[(-1)^k-1]}{2\sqrt{\xi_0^2-1}\,P_k'(\xi_0)\,\lambda}\int_0^{\varphi} P_k(\cos\theta)\sqrt{\xi_0^2-\cos^2\theta}\,\sin\theta\,d\theta.$$

Thus for all k = 1, 2, 3, . . .

$$a_{2k} = 0$$

and

$$a_{2k-1} = -\frac{(4k-1)k_0 q}{\lambda\sqrt{\xi_0^2-1}\,P_{2k-1}'(\xi_0)}\int_0^{\varphi} P_{2k-1}(\cos\theta)\sqrt{\xi_0^2-\cos^2\theta}\,\sin\theta\,d\theta.$$

After substituting $\cos\theta = t$, we obtain that

$$a_{2k-1} = \frac{(4k-1)k_0 q}{\lambda\sqrt{\xi_0^2-1}\,P_{2k-1}'(\xi_0)}\int_1^{\cos\varphi} P_{2k-1}(t)\sqrt{\xi_0^2-t^2}\,dt.$$

To find coefficient a_0 without losing generality, we assume that the temperature in the equatorial section of the ellipsoid is equal to zero [2]. Then

$$a_0 = 0.$$

We finally write the formula of the temperature field of an ellipsoid with two contact areas in the following manner:

$$T(\xi, \nu) = \frac{qk_0}{\lambda\sqrt{\xi_0^2 - 1}} \sum_{k=1}^{\infty} b_{2k-1} P_{2k-1}(\xi) P_{2k-1}(\nu),$$

where

$$b_{2k-1} = \frac{4k-1}{P'_{2k-1}(\xi_0)} \int_1^{\cos\varphi} P_{2k-1}(t)\sqrt{\xi_0^2 - t^2}\, dt;$$

$$k_0 = \sqrt{h_e^2 - b_e^2} \text{ and } \xi_0 = \frac{h_e}{\sqrt{h_e^2 - b_e^2}}.$$

To simplify the calculations we assume the temperature of the contact area to be equal to the arithmetic mean of the temperatures in the center and at the edges of the area [2]:

$$T_{av} = \frac{T(\xi_0, 1) + T(\xi_0, \cos\varphi)}{2}.$$

The temperature difference between the upper and lower contact areas is equal to

$$\Delta T = 2T_{av}.$$

The thermal resistance of the ellipsoid with two contact areas is equal to

$$R_e = \frac{\Delta T}{q\pi a^2} = \frac{2T_{av}}{q\pi a^2}$$

or finally

$$R_e = \frac{1}{\pi\lambda k_0 \xi_0^2 \sqrt{\xi_0^2 - 1}\sin^2\varphi} \sum_{k=1}^{\infty} b_{2k-1}\{P_{2k-1}(\xi_0)[1 + P_{2k-1}(\cos\varphi)]\}.$$

We determine the radius of the contact spot by Hertz' formula with substitution of the value for the total force P_t [2], assuming that transverse deformation of the ellipsoid is absent and that force P_t creates only longitudinal deformation (Poisson's ratio $\mu = 0$) [4]:

$$a = 0.93\rho \sqrt[3]{\frac{p_{sp}}{E} \Big/ (1 - m)^2}.$$

For the model we selected with orderly packing of the elements (ellipsoids), viz., simple cubic packing, the porosity of the plate m = 0.476. After substituting the values of m and ρ, we obtain finally

$$a = 1.43 \frac{b_e^2}{h_e} \sqrt[3]{\frac{p_{sp}}{E}} = 1.43\varkappa^2 h_e \sqrt[3]{\frac{p_{sp}}{E}}.$$

The contact resistance of one plate is equal to

$$R_{cont.\ pl} = \frac{R_e}{N_1}, \qquad (6)$$

where

$$N_1 = \frac{N_e}{F_{pl}} \text{ and } N_e = \frac{F_{pl}}{4b_e^2} = \frac{F_{pl}}{4\varkappa^2 h_e^2}.$$

After substituting N_1 into (6) we obtain

$$R_{cont.\ pl} = 4\varkappa^2 h_e^2 R_e.$$

The total contact resistance of the packet will be

$$R_{cont} = 4\varkappa^2 h_e^2 (n-1) R_e.$$

A comparison of the data on the thermal resistance of multicontact packets calculated on the basis of the known theoretical models in [1] and [5] and by Eq. (7) with the experimental data is given in Fig. 3. As we see from Fig. 3, the considerable divergence (from 3 to 8 fold) with the experiment is observed in calculations by the theoretical model in [1]. With respect to the theoretical model in [5] the best agreement was obtained for large values of the specific force (more than 20 MN/m^2); for small values of the specific force (about 5 MN/m^2) the discrepency with the experiments is more than twofold.

The best agreement with the experiment is observed in calculating the thermal resistances of the packets by Eq. (7) for average values of $\varkappa$, indicated in the graphs (curves a and b, Fig. 3).

Figure 4 shows $\varkappa$ as a function of p_{sp} for two packets obtained from experiments with conversion by Eq. (7). The packets were made up of plates of stainless steel Kh18N10T-M with a thickness of 0.1 and 0.2 mm with a surface finish of ∇10 and ∇9, respectively. The surfaces of the plates were degreased during assembly.

An analysis of the character of change of $\varkappa$ (Fig. 4) as a function of the specific force for multicontact packets vindicates our choice of the selected diagram of the heat flow in the packets and proposed method of calculating the thermal resistance of packets. Actually, on compressing a multicontact packet of metal plates the number of contacts participating in heat transfer increases (along with a slight increase of each contact area) as the force increases. According to the selected diagram of heat flow in the packet, the heat is redistributed as the number of contacts increases and the number of ellipsoids in the plate increases, but the ellipsoids themselves contract, i.e., $\varkappa$ decreases. Finally, the plates with a greater thickness but with almost the same surface finish, $\varkappa = b_e/h_e$ is smaller for the same pressure, since b_e, which is determined by the distance between contacts, i.e., the surface finish, remains practically unchanged.

Of unquestionable interest is the finding of the analytical dependence of $\varkappa$ on the specific force, thickness of the plates of the packet, mechanical properties of the contacting materials, and their surface quality. It is proposed to do this in subsequent studies.

NOTATION

δ_{pl}, F_{pl}	are the plate thickness and area;
h_e, b_e	are the semimajor and semiminor axes of the ellipsoid;
a	is the radius of the contact area;
$\rho = b_e^2/h_e$	is the radius of the curvature at the contact point of the ellipsoid;
$\varkappa = b_e/h_e$	is the coefficient of contraction of the ellipsoid;
N_1	is the number of contact spots per unit nominal surface of contact of the plates;
N_e	is the number of ellipsoids in the plate;
n	is the number of plates in the packet;
$\bar{E} = 2E_1E_2/(E_1+E_2)$, $\bar{\lambda} = 2\lambda_1\lambda_2/(\lambda_1+\lambda_2)$	are the reduced modulus of elasticity and reduced coefficient of thermal conductivity of the contacting materials with consideration of the boundary temperatures T_1 and T_2, respectively;
q	is the specific heat flux;
R_e, $R_{cont.pl}$, R_{cont}	are the thermal resistances of the ellipsoid, plate, and packet;
P_t, p_{sp}	are the total and specific compressive force of the packet.

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